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A 'c Policy' Batch Service Queueing System under Bernoulli Scheduled Vacation and Manual Service

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ABSTRACT

The discrete time single arrival and single-batch service queue under policy c is described in this study together with reneging and vacation interruptionThe system activates only when the queue size reaches a predetermined point c. When the queue size reaches c, the system activates and server serves all the units in a batch. After a batch completion epoch, if there is queue size less than c, either server switches to manual service with the probability p, or server takes vacation with probability $(1-p)$. The same continues once the system starts manual service or the server goes on vacation until the queue size reaches c.Reneging of the customer is taken into consideration prior to the queue size reaching c when the server on vacation. The inter-arrival and service time are presumed to be independent and geometrically distributed.The steady-state probabilities of the model is expressed in terms of parameters.The measures such as average reneging rate, expected queue length, expected waiting time, probability of busy period and optimization are discovered.The particular cases are studied and numerical are done for the model.

Key words and Phrases:*Discrete time-arrival and service, Single and Batch Service Queue, c Policy,Reneging,Vacation interruption, Geometric distribution.* .

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1 Introduction

Since reneging has drawn the attention of numerous researchers and has found use in business, medical emergencies, inventory systems, the construction industry, etc., it plays a significant role in the performance and revenue issues of the queueing model. Loss of customers might occur when the system is idle (Laxmi et al.(2013)) or impatience of the customer to wait in the queue for a long time because of the particular discipline of the system would encourage them to renege. When the expected reneging rate is found, it is possible to make changes in the system to control the lost revenue(Goswami(2014)). Haight(1959) considered a person in the queue reneging when the waiting time exceeded his available time and investigated how this would affect other units in the system. Ancker(1963a) investigated a limited capacity $M/M/1$ queue including reneging. Robert(1979) considered a single-channel queueing model with reneging whereas Haghighi(1986) considered a multichannel $M/M/c$ queue with reneging and balking and derived its steady-state solution. In 2012, Rakesh derived the steady-state solution for finite capacity $M/M/1$ queue with bernoulli reneging schedule.

Vacation interruption queueing model have been studied since 1980's by many researchers such as Doshi(1986), Takagi(1991), Tian and Zhang(2006), so on. Vacation time of the server will be used for his personal needs, or extra pending works in the system, or helping co-workers of the system so on. Jau Chuan ke(2010) in, showed a detailed study of the vacation interruption model. Vijayalaxmi(2013) obtained the performance measures for Markovian queue model with vacation interruption under N policy with reneging and balking. In 2020, Chakravarthy studied the $MAP/PH/1$ queueing model with reneging of the customer when the server in the vacation, server repair, server breakdown.

One of the flexible and profitable methods in a system is applying both manual and batch service which decreases reneging and traffic intensity in the model. In 1998, Baburaj and Manohar summed up a model of single and batch service queue with accessibility. Baburaj and Rekha (2013) looked into the steady-state solution of discrete-time model with manual and batch service discipline along with Bernoulli schedule. In 2009, Goswami and Samanta described the model of two types of service methods giving accessibility to the batch. Here, in this paper the two service rules; batch service and manual service with reneging and vacation interruption is considered.

The discrete-time batch service queueing model with manual service and vacation time shown in this paper using the Bernoulli schedule. When the queue size hits c, the system starts service in that slot and serves all units in a batch. If the server discovers that there are more units in the queue than c, he continues to serve in batches. According to a bernoulli schedule, the server will go on vacation or enter manual service at a batch service completion epoch. The server either switches to manual service with probability p or takes a vacation with probability $(1-p)$ if the queue size is less than c. Before the queue size exceeds c, when the server on vacation, the probability of customers reneging is also taken into account.The shift operator method and the recursive method are using to evaluate the steady-state probabilities. The model will be thoroughly examined in the following manner. Numerical examples are using to demonstrate the impact of the predetermined limit c on the expected length of the queue, the length of the wait, and the cost.

2 Model Description

Discrete time 'c policy' batch service model with manual service and vacation under the bernoulli schedule executes as follows,

- In every consecutive slot, customers who enter for service are observed as independent identically distributed random variables which will be follow geometric distribution with parameter λ and hold back in the line until the its size comes to a predetermined control limit c.

- When the length of waiting line hits c, the server activates the system and all units are served by batches. The required time for the services is observed as a geometric random variable with rate μ_1. After a batch service completion epoch, if the queue size exceeds the point c, the server carries on with the batch service itself.

- If the queue size drops below c, server execute Bernoulli schedule procedure such as, with probability, p server opts the manual service(One customer served at a time), where the service time will be a geometric distributed random variable with the rate μ_2, or with probability $(1-p)$ server takes vacations with the rate β until the queue size reaches c. When the server in manual service, the arriving customers joins the queue and waits for the service.The reneging may occur in every server's vacation time slot, which is observed as random geometric variables with rate α. let us denote the reneging rate by $r_i$where i is the number of customers waiting in the queue. The arrival or departure of any customers are independent to each other hence the average reneging will be considered as $r_i = i\alpha$,$r_0 = 0$,$r_i = 0 \;\; for \;\; i \geqslant c$.
  The parameters are,

Parameter	Description
λ	Arrival rate
α	Reneging rate
μ_1	Batch service rate
μ_2	Manual service rate
β	Vacation rate
r_i	Average reneging

3 Analysis

The amount of units in the waiting line at time t is denoted by N(t).

- P(0,n,t-) denote the probability of the server on vacation state in the system and n customers in the queue at time $t-$, $n = 0, 1, 2...c-1$

- P(1,n,t-) denotes the probability of server is busy with batch service and n customers waiting in the queue at time $t-$,$n = 0, 1...$
- The probability that the server is busy providing manual service and there are n clients in the queue at time $t-$ is indicated by the expression P(2,n,t-), $n = 0, 1, 2..c-1$.

Here $t-$ and $(t+1)-$ are potential time epochs of arrivals and took them as t and $(t+1)$ for make analysis simple.

Hence, as $t \to \infty$,the probabilities will be:

$P(0, n, t) \to W_n, \qquad n = 0, 1, 2...c-1$

$P(1, n, t) \to P_n, \qquad n = 0, 1...$

$P(2, n, t) \to Q_n, \qquad n = 0, 1, 2..c-1$ and

the below model represents the transition diagram of batch service queueing model including manual service and vacation under the Bernoulli schedule and reneging.

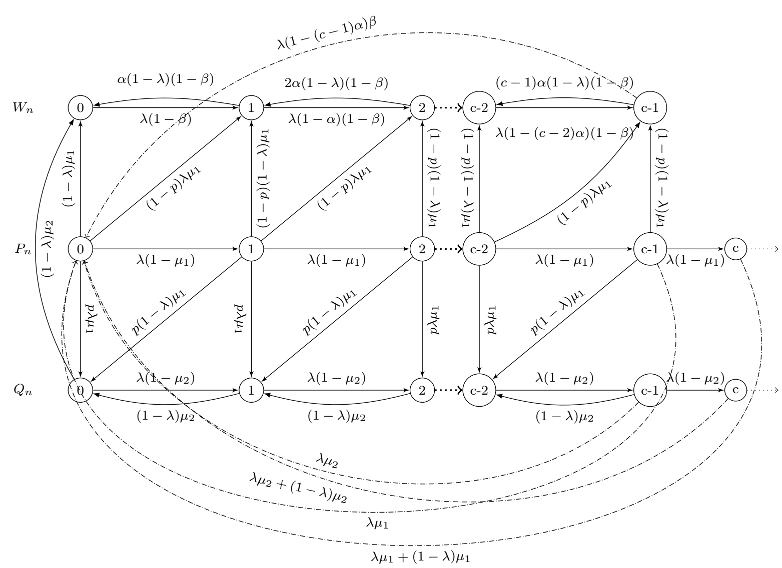

Figure-1: Transition diagram

The steady state probabilities are,

$$\begin{aligned}
\lambda(1-\beta)W_0 &= \alpha(1-\lambda)(1-\beta)W_1+\mu_2(1-\lambda)Q_0 \\
&\quad +(1-\lambda)\mu_1 P_0 \qquad (3.1)\\
(1-\beta)(\lambda+n\alpha-2n\lambda\alpha)W_n &= (1-\lambda)(n+1)\alpha(1-\beta)W_{n+1} \\
&\quad +\lambda(1-(n-1)\alpha)(1-\beta)W_{n-1} \\
&\quad +(1-p)\mu_1(1-\lambda)P_n+(1-p)\lambda\mu_1 P_{n-1}, \\
&\qquad n=1,2,3...(c-2) \qquad (3.2)\\
\left(\begin{matrix}\lambda\beta+(c-1)\alpha \\ (1-(\lambda+\beta))\end{matrix}\right)W_{c-1} &= \lambda(1-(c-2)\alpha)(1-\beta)W_{c-2} \\
&\quad +(1-p)\mu_1(1-\lambda)P_{c-1} \\
&\quad +(1-p)\lambda\mu_1 P_{c-2} \qquad (3.3)\\
(\lambda+\mu_1-\lambda\mu_1)P_0 &= \lambda(1-(c-1)\alpha)\beta W_{c-1}+\lambda\mu_2\sum_{n=c-1}^{\infty}Q_n \\
&\quad +\mu_2(1-\lambda)\sum_{n=c}^{\infty}Q_n+\mu_1(1-\lambda)\sum_{n=c}^{\infty}P_n \\
&\quad +\lambda\mu_1\sum_{n=c-1}^{\infty}P_n \qquad (3.4)\\
(\lambda+\mu_1-\lambda\mu_1)P_n &= \lambda(1-\mu_1)P_{n-1}, \qquad n=1,2,3... \qquad (3.5)
\end{aligned}$$

$$\begin{aligned}
(\lambda+\mu_2-2\lambda\mu_2)Q_0 &= p\lambda\mu_1 P_0+p\mu_1(1-\lambda)P_1 \\
&\quad +(1-\lambda)\mu_2 Q_1 \qquad (3.6)
\end{aligned}$$

$$\begin{aligned}
(\lambda+\mu_2-2\lambda\mu_2)Q_n &= p\mu_1\lambda P_n+p(1-\lambda)\mu_1 P_{n+1} \\
&\quad \lambda+(1-\mu_2)Q_{n-1}+(1-\lambda)\mu_2 Q_{n+1}, \\
&\qquad n=1,2,...(c-2) \qquad (3.7)\\
(\lambda+\mu_2-\lambda\mu_2)Q_n &= \lambda(1-\mu_2)Q_{n-1} \qquad n=c-1,c,c+1... (3.8)
\end{aligned}$$

From the equation (3.5), we obtain,

$$P_n = a_1^n P_0, \qquad n=1,2,... \qquad (3.9)$$

where

$$a_1 = \frac{\lambda(1-\mu_1)}{\lambda(1-\mu_1)+\mu_1}$$

Let us define some coefficients,

$a_2 = \frac{\lambda(1-\mu_2)}{\mu_2(1-\lambda)}$, $a_3 = \frac{\mu_2(1-\lambda)}{\lambda+\mu_2-2\lambda\mu_2}$, $a_4 = \frac{\mu_1(1-\lambda)}{\lambda+\mu_2-2\lambda\mu_2}$,
$a_5 = \frac{\mu_1\lambda}{\lambda+\mu_2-2\lambda\mu_2}$, $a_6 = \frac{\mu_1(1-\lambda)}{\lambda+\mu_2-\lambda\mu_2}$, $a_7 = \frac{\lambda(1-\mu_2)}{\lambda+\mu_2-\lambda\mu_2}$,
$a_8 = \frac{\mu_1\lambda}{\lambda+\mu_2-\lambda\mu_2}$.

The equation (3.7),reflects in the form of differential equation, hence,

$$\begin{aligned} Q_n &= (\frac{\lambda}{1-\lambda})^n(\frac{1-\mu_2}{\mu_2})^n Q_0 - \frac{p\mu_1\lambda a_1^n + p(1-\lambda)\mu_1 a_1^{n+1}}{F(a_1)}P_0 \\ &= a_2^n Q_0 - \frac{p\mu_1 a_1^n[\lambda+(1-\lambda)a_1]}{F(a_1)}P_0 \end{aligned} \tag{3.10}$$

where a_2 is the unique root of the auxiliary equation, $F(z) = (1-\lambda)\mu_2 z^2 - (\lambda + \mu_2 - 2\lambda\mu_2)z + \lambda(1-\mu_2)$

From equation (3.6),

$$Q_0 = g(\mu_1,\mu_2)P_0 \tag{3.11}$$

where

$$g(\mu_1,\mu_2) = \frac{p}{1-a_2a_3}\left\{(a_1a_4+a_5) - \frac{\mu_1 a_1 a_3[\lambda+(1-\lambda)a_1]}{F(a_1)}\right\}$$

using the expression of Q_0 in Q_n,we get

$$Q_n = h(\mu_1,\mu_2,n)P_0, \qquad n = 1,2,3...c-2 \tag{3.12}$$

where,

$$h(\mu_1,\mu_2,n) = \frac{a_2^n(a_1a_4+a_5)p}{1-a_2a_3} - p\mu_1\left\{\frac{\lambda+(1-\lambda)a_1}{F(a_1)}\right\}\left\{\frac{a_2^n a_1 a_3}{1-a_2a_3} - a_1^n\right\}$$

Similarly equation(3.8) will be,

$$Q_n = a_7^n h(\mu_1,\mu_2,c-2)P_0, \qquad n = c-1, c, c+1... \tag{3.13}$$

Equation (3.4) can be written as,

$$
\begin{aligned}
W_{c-1} &= \big(e_1 - h(\mu_1,\mu_2,c-2)\frac{a_7^{c-1}}{1-a_7}[e_2 + e_3 a_7] \\
&\quad -\frac{a_1^{c-1}}{\beta(1-a_1)}[e_{(4,c-1)}a_1 + e_{(5,c-1)}]\big)P_0 \\
&= \xi_{c-1}P_0 \qquad (3.14)
\end{aligned}
$$

Where,

$e_1 = \frac{\lambda+\mu_1-\lambda\mu_1}{\lambda(1-(c-1)\alpha)\beta}$, $e_2 = \frac{\lambda\mu_2}{\lambda(1-(c-1)\alpha)\beta}$, $e_3 = \frac{\mu_2(1-\lambda)}{\lambda(1-(c-1)\alpha)\beta}$,

$e_{(4,c-1)} = \frac{\mu_1(1-\lambda)}{\lambda(1-(c-1)\alpha)}$, $e_{(5,c-1)} = \frac{\lambda\mu_1}{\lambda(1-(c-1)\alpha)}$

Hence, equation(3.3) yields,

$$
\begin{aligned}
W_{c-2} &= \left\{e_5\xi_{c-1} - \frac{a_1^{c-2}(1-p)}{(1-\beta)}[e_{(4,c-2)}a_1 + e_{(5,c-2)}]\right\}P_0 \\
&= \xi_{c-2}P_0 \qquad (3.15)
\end{aligned}
$$

where, $e_5 = \frac{\lambda\beta+(c-1)\alpha(1-(\lambda+\beta))}{\lambda(1-(c-2)\alpha)(1-\beta)}$

Solving equations (3.2) recursively gives,

$$
\begin{aligned}
W_n &= \xi_n P_0 \\
&= \left\{k_{(1,n+1,n)}\xi_{n+1} - k_{(2,n+2,n)}\xi_{n+2} - \frac{(1-p)a_1^n}{(1-a_1)(1-\beta)}[e_{(4,n)}a_1 + e_{(5,n)}]\right\} \\
&\qquad n = c-3, c-4, ..3, 2, 1. \qquad (3.16)
\end{aligned}
$$

where,

$k_{(1,n+1,n)} = \frac{\lambda+(n+1)\alpha-2(n+1)\alpha\lambda}{\lambda(1-n\alpha)}$

$k_{(2,n+2,n)} = \frac{(1-\lambda)(n+2)\alpha}{\lambda(1-n\alpha)}$

Solving equation(3.1) gives,

$$
W_0 = \frac{1-\lambda}{\lambda}\left\{\alpha\xi_1 + \frac{\mu_2}{(1-\beta)}g(\mu_1,\mu_2) + \frac{\mu_1}{(1-\beta)}\right\}P_0 \qquad (3.17)
$$

Finally, P_0 can be obtained from normalizing condition,

$$\begin{aligned} P_0 &= \Bigg[1+\frac{a_1}{1-a_1}+g(\mu_1,\mu_2)+\frac{a_7^{c-1}}{1-a_7}h(\mu_1,\mu_2,c-2)+\frac{(a_1a_4+a_5)p}{1-a_2a_3}\frac{a_2(1-a_2^{c-2})}{1-a_2} \\ &\quad -p\mu_1\frac{\lambda+(1-\lambda)a_1}{F(a_1)}\left\{\frac{a_1a_3a_2(1-a_2^{c-2})}{(1-a_2)(1-a_2a_3)}-\frac{a_1(1-a_1^{c-2})}{1-a_1}\right\}+\xi_{c-1} \\ &\quad +\xi_{c-2}+\sum_{n=1}^{c-3}\xi_n+\frac{1-\lambda}{\lambda}\big(\alpha\xi_1+\frac{\mu_2}{(1-\beta)}g(\mu_1,\mu_2)+\frac{\mu_1}{(1-\beta)}\big)\Bigg]^{-1} \end{aligned} \tag{3.18}$$

4 Performance Measures and Particular Cases

4.1 Average Reneging Rate

When the system is in an idle state, and if there are k customers waiting for the service, then the customers might show the tendency of reneging. Due to the impatience of the customer, any customer in the queue can renege, with the reneging rate is $k\alpha$. The average reneging is expressed as,

$$A.R = \sum_{n=1}^{c-1} n\alpha\xi_n P_0 \tag{4.1}$$

4.2 Expected Queue length and Waiting time of the model

The queue length of this model is estimated using four states of system as follows.

$$\begin{aligned} L_q &= \Bigg\{\frac{a_1}{(1-a_1)^2}+\frac{a_1a_4p}{1-a_2a_3}\Big[\frac{1-a_2^{c-2}-(c-2)a_2^{c-2}(1-a_2)}{(1-a_2)^2}\Big] \\ &\quad +\left(\frac{\lambda+(1-\lambda)a_1}{F(a_1)(1-a_2a_3)}\right)\Big\{p\mu_1a_1a_3\Big(\frac{1-a_2^{c-2}-(c-2)a_2^{c-2}(1-a_2)}{(1-a_2)^2}\Big) \\ &\quad -p\mu_1\Big(\frac{1-a_1^{c-2}-(c-2)a_1^{c-2}(1-a_1)}{(1-a_1)^2}\Big)\Big\}+(c-1)j(\mu_1,\mu_2)+\sum_{n=1}^{c-1}n\xi_n\Bigg\}P_0 \end{aligned}$$

And the expected waiting time of customer is expressed as,

$$W_q = L_q/\lambda \tag{4.2}$$

4.3 Probability of Busy period

In this model, Busy period of server measured from batch and manual service.Hence the probability of busy period of server is,

$$
\begin{aligned}
P(B) &= \sum_{n=1}^{\infty} P_n + \sum_{n=1}^{c-1} Q_n \\
&= \left\{ \frac{a_1}{(1-a_1)} + \frac{a_1 a_4 p}{1-a_2 a_3} \frac{a_2(1-a_2^{c-2})}{1-a_2} - \frac{\lambda+(1-\lambda)a_1}{F(a_1)(1-a_2a_3)} \left\{ \frac{p\mu_1 a_1 a_3 a_2 (1-a_2^{c-2})}{1-a_2} \right.\right. \\
&\quad \left.\left. - \frac{p\mu_1 a_1 (1-a_1^{c-2})}{1-a_1} \right\} + j(\mu_1,\mu_2) \right\} P_0
\end{aligned}
$$

4.4 Optimization

The costs involved in this model are holding cost per unit(C_h), the cost required to set up batch service(C_0), the value need for service when the server in batch service(C_1), and manual service(C_2). Using these costs we can define the total cost function,

$$
\begin{aligned}
Cost &= C_h L_q + C_0 \sum_{n=0}^{\infty} P_n + C_2 \sum_{n=0}^{\infty} (n+1) Q_n + C_1 \sum_{n=c}^{\infty} P_n \\
&\quad + \sum_{n=0}^{c-1} (C_1 + pnC_2) P_n \qquad (4.3)
\end{aligned}
$$

Our objective is to find the value of the lower limit 'c' such that the total cost function is minimum.

4.5 Particular Cases:

4.5.1 Case (i):

When $p = 0$ and $\alpha = 0$ (without manual service and reneging of the customer),the steady state conditions are,

$$\begin{aligned}
\lambda(1-\beta)W_0 &= (1-\lambda)\mu_1 P_0 && (4.4)\\
(1-\beta)\lambda W_n &= \lambda(1-\beta)W_{n-1} + \mu_1(1-\lambda)P_n + \lambda\mu_1 P_{n-1}, &&\\
& \qquad\qquad n = 1,2,3...(c-2) && (4.5)\\
\lambda\beta W_{c-1} &= \lambda(1-\beta)W_{c-2} + \mu_1(1-\lambda)P_{c-1} + \lambda\mu_1 P_{c-2} && (4.6)\\
(\lambda+\mu_1-\lambda\mu_1)P_0 &= \lambda\beta W_{c-1} + \mu_1(1-\lambda)\sum_{n=c}^{\infty} P_n + \lambda\mu_1 \sum_{n=c-1}^{\infty} P_n && (4.7)\\
(\lambda+\mu_1-\lambda\mu_1)P_n &= \lambda(1-\mu_1)P_{n-1}, \qquad n = 1,2,3... && (4.8)\\
& && (4.9)
\end{aligned}$$

This model consists of two states,vacation and batch service. Hence it represent a single arrival, batch service queueing model with server vacation under policy 'c'.

4.5.2 Case (ii):

When $p = 1$ and $\alpha = 0$ are used in the steady state model equations, the model demonstrates batch service and manual service characteristics occurs under policy 'c'.

Case (iii):

When $\alpha = 0$,$\beta = 0$ and there is no control over starting of service(server starts manual service when the first customer enters the system and continues the same until queue size reaches c and switches his service to the batch service right after the queue size is c.At the end of a batch service, if the server finds queue size less than control limit, then the server works based on Bernoulli schedule.With probability $(1-p)$ server moves with batch service otherwise with probability p server chooses manual service), then the steady state probabilities turns as follows.

$$\lambda W_0 = \mu_1(1-\lambda)P_0 + \mu_2(1-\lambda)Q_0 \tag{4.10}$$

$$(\lambda+\mu_1-\lambda\mu_1)P_0 = \mu_1(1-\lambda)\sum_{n=c}^{\infty}P_n + \mu_1(1-\lambda)(1-p)\sum_{n=1}^{c-1}P_n + \mu_1\lambda(1-p)\sum_{n=0}^{c-2}P_n + \lambda\mu_1\sum_{n=c-1}^{\infty}P_n + \lambda\mu_2 Q_{c-1}$$

$$(\lambda+\mu_1-\lambda\mu_1)P_n = \lambda(1-\mu_1)P_{n-1}, \qquad n=1,2,3... \tag{4.11}$$

$$(\lambda+\mu_2-2\lambda\mu_2)Q_0 = \lambda W_0 + p\lambda\mu_1 P_0 + p\mu_1(1-\lambda)P_1 + (1-\lambda)\mu_2 Q_1$$

$$(\lambda+\mu_2-2\lambda\mu_2)Q_n = p\mu_1\lambda P_n + p(1-\lambda)\mu_1 P_{n+1} + \lambda(1-\mu_2)Q_{n-1} + (1-\lambda)\mu_2 Q_{n+1}, \qquad n=1,2,...(c-2) \tag{4.12}$$

$$(\lambda+\mu_2-2\lambda\mu_2)Q_{c-1} = p\mu_1\lambda P_{c-1} + (1-\lambda)\mu_1 P_c + \lambda(1-\mu_2)Q_{c-2} \tag{4.13}$$

These are the steady-state probabilities of Bernoulli scheduled manual and batch service model.

Case (iv):

When the value of c strikes 1, then there is no importance for reneging concept, and hence entering customers are all served in batches. So there will be no significance of manual service and Bernoulli schedule. When the queue size becomes zero, the server goes on vacation and takes multiple vacations until a unit appears in the system. The steady-state probability of this system will be,

$$\lambda\beta W_0 = \mu(1-\lambda)P_0$$

$$(\lambda+\mu_1-\lambda\mu_1)P_0 = \mu_1(1-\lambda)\sum_{n=1}^{\infty}P_n + \lambda\mu_1\sum_{n=0}^{\infty}P_n + \lambda\beta W_0 \tag{4.14}$$

$$(\lambda+\mu_1-\lambda\mu_1)P_n = \lambda(1-\mu_1)P_{n-1}, \qquad n=1,2,3... \tag{4.15}$$

(4.16)

solving the above equations results in,

$$\begin{aligned} W_0 &= \frac{\mu_1(1-\lambda)}{\lambda\beta} P_0 \\ P_n &= a_1^n P_0 \end{aligned}$$

where $a_1 = \frac{\lambda(1-\mu_1)}{\lambda+\mu_1-\lambda\mu_1}$
and, hence P_0 is,

$$P_0 = \left\{ \frac{\mu_1(1-\lambda)}{\lambda\beta} + 1 + \frac{\lambda(1-\mu_1)}{\mu_1} \right\}^{-1}$$

Which is the steady state probabilities of single arrival- batch service with vacation model.

5 Numerical Results

Assuming different values for the parameters in the model, the effect of c on L_q, the effect of c on $Cost^*$, and the effect of p on waiting times are picturized as below.

- Figure-I represents the effects of lower control limit c on the Total cost ($Cost^*$) by taking $\lambda = .2, \mu 1 = .45, \mu 2 = .28, \alpha = 0.01$ and $\beta = .18$. Assuming $C_h = 150, C_0 = 200, C_1 = 100, C_2 = 80$, for the first graph and rest are given in the Figure-I.

- Figure-II represents the effect of lower limit c on the Total cost ($Cost^*$) ,for $C_h = 50, C_0 = 20, C_1 = 10, C_2 = 15, C_3 = 20$.Assuming $\lambda = .2, \mu 1 = .65, \mu 2 = .51$, and $\beta = .28$ for the first graph and $\lambda = .4, \mu 1 = .83, \mu 2 = .45$, and $\beta = .78$ as given in the Figure-II.

- Figure-III represents the effects of lower control limit c on the waiting time of the customer.

Figure-I:Effect of c on the $Cost^*$

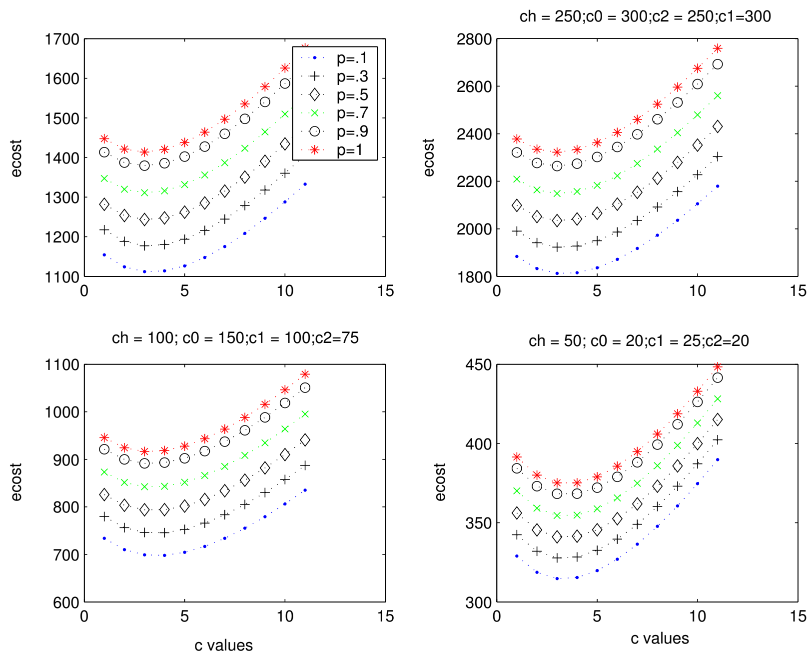

Figure-II Effect of c on the $Cost^*$

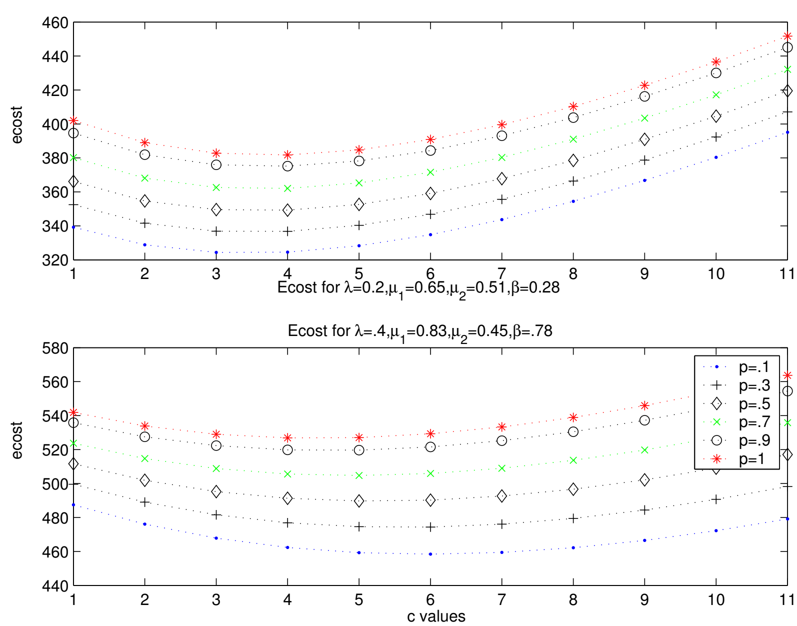

Figure-III:Effect of c on the W_q

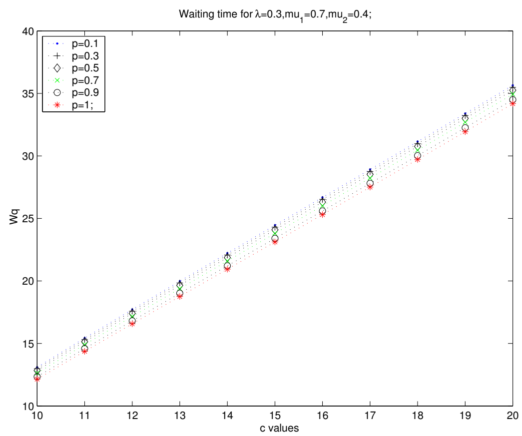

Figure-I shows the pattern of the cost curve for different values of cost parameter and probability. The optimum value of the lower limit c will be obtained by taking the same values for the system parameters and different values for the cost parameters as shown in the figure. Figure-II characterizes the changes in the optimum value of c as the system parameters changes. Figure-III depicts the increasing waiting time of the customer as the lower limit increases for different values of probability.

6 Conclusion

In this paper, a Bernoulli scheduled manual service and vacation of server included in a single arrival batch service system under policy c. In terms of cost and time, serving customers in batches or providing manual service in accordance with the limit 'c' will be more cost-effective. When the queue size surpasses the control limit, the server starts batch service and provides all units in one batch. The server performs Bernoulli scheduled manual service and vacation when the queue size is below the limit.Because the cost function is reliant on c, lowering cost yields the best c value. The probability of a Bernoulli schedule and the selection of control limits play a large effect on cost, as seen in the numerical picture.

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